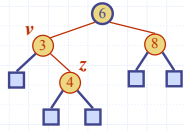
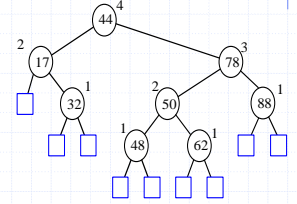


AVL Trees



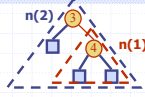
AVL Tree Definition (§9.2)

- ◆ AVL trees are **balanced**.
- ◆ An AVL Tree is a **binary search tree** such that for every internal node v of T , the **heights of the children of v can differ by at most 1**.



An example of an AVL tree where the heights are shown next to the nodes:

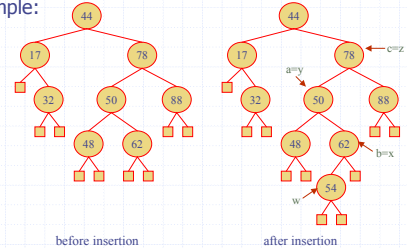
Height of an AVL Tree



- ◆ **Fact:** The **height** of an AVL tree storing n keys is $O(\log n)$.
- ◆ **Proof:** Let us bound $n(h)$: the minimum number of internal nodes of an AVL tree of height h .
- ◆ We easily see that $n(1) = 1$ and $n(2) = 2$
- ◆ For $n > 2$, an AVL tree of height h contains the root node, one AVL subtree of height $n-1$ and another of height $n-2$.
- ◆ That is, $n(h) = 1 + n(h-1) + n(h-2)$
- ◆ Knowing $n(h-1) > n(h-2)$, we get $n(h) > 2n(h-2)$. So $n(h) > 2n(h-2)$, $n(h) > 4n(h-4)$, $n(h) > 8n(h-6)$, ... (by induction), $n(h) > 2^n n(h-2i)$
- ◆ Solving the base case we get: $n(h) > 2^{h/2-1}$
- ◆ Taking logarithms: $h < 2 \log n(h) + 2$
- ◆ Thus the height of an AVL tree is $O(\log n)$

Insertion in an AVL Tree

- ◆ Insertion is as in a binary search tree
- ◆ Always done by expanding an external node.
- ◆ Example:

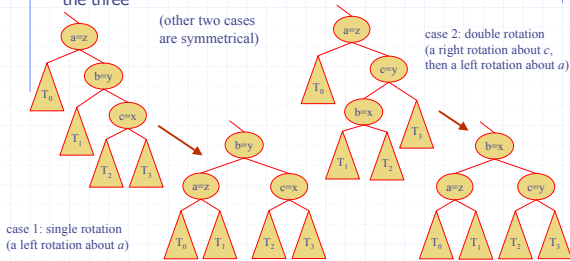


before insertion

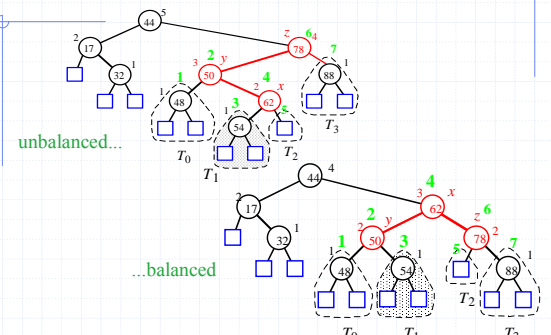
after insertion

Trinode Restructuring

- ◆ let (a, b, c) be an inorder listing of x, y, z
- ◆ perform the rotations needed to make b the topmost node of the three



Insertion Example, continued

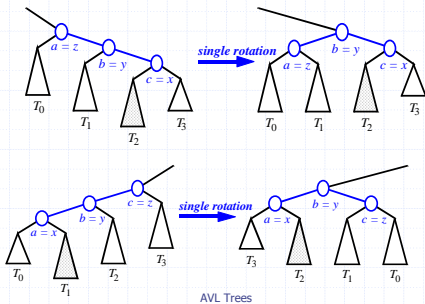


unbalanced...

...balanced

Restructuring (as Single Rotations)

Single Rotations:

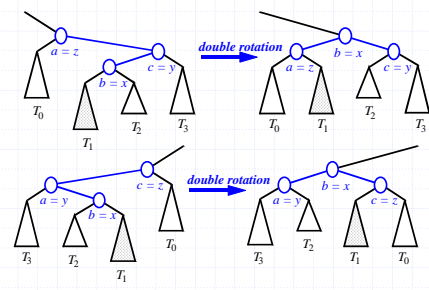


AVL Trees

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Restructuring (as Double Rotations)

double rotations:

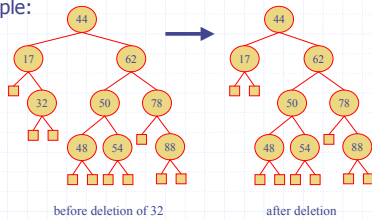


AVL Trees

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Removal in an AVL Tree

- ◆ Removal begins as in a binary search tree, which means the node removed will become an empty external node. Its parent, w , may cause an imbalance.
- ◆ Example:



before deletion of 32

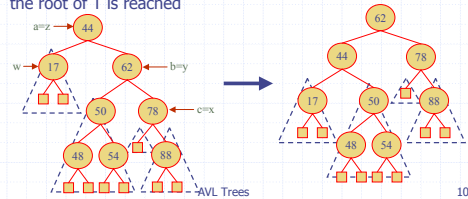
after deletion

AVL Trees

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Rebalancing after a Removal

- ◆ Let z be the first unbalanced node encountered while travelling up the tree from w . Also, let y be the child of z with the larger height, and let x be the child of y with the larger height.
- ◆ We perform `restructure(x)` to restore balance at z .
- ◆ As this restructuring may upset the balance of another node higher in the tree, we must continue checking for balance until the root of T is reached



AVL Trees

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Running Times for AVL Trees

- ◆ a single restructure is $O(1)$
 - using a linked-structure binary tree
- ◆ find is $O(\log n)$
 - height of tree is $O(\log n)$, no restructures needed
- ◆ insert is $O(\log n)$
 - initial find is $O(\log n)$
 - Restructuring up the tree, maintaining heights is $O(\log n)$
- ◆ remove is $O(\log n)$
 - initial find is $O(\log n)$
 - Restructuring up the tree, maintaining heights is $O(\log n)$



AVL Trees

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